MA 573 Note for Lecture Note 3

Definition of derivative by limit:

If is small enough, then the definition of derivative will become:

is finite difference operator ().

* If , then is forward finite difference operator ().
* If , then is backward finite difference operator ().

The central finite difference operator () is the average of and :

By Taylor’s Theorem, which is a theorem gives an approximation of -times differential function by a th-order polynomial, both and have convergence order 1, such that, given or when is small,

And,

has convergence order 2 because it yields a more accurate approximation by the 2nd-order polynomial or quadratic polynomial.

Proof:

Take the limit of the left term, and by *l’Hopitals* Rule, we have the relationship, such that

This means that everything goes back to the original, such that

There is a proof of , by continuity, we can use the similar idea, such that

For

Since based on a graph that is about ,

图片包含 激光, 场景, 飞行, 男人

描述已自动生成

The function is continuous, and by the continuity and the condition that is small, we can also conclude that

Therefore,

Reference Information:

表格

描述已自动生成

<https://en.wikipedia.org/wiki/Finite_difference>

<https://en.wikipedia.org/wiki/Taylor%27s_theorem>

<https://www.youtube.com/watch?v=JMttb5AsARE>